

Evaluating Definite Integrals with Geometry

Sketch the region whose area is given by the definite integral. Use geometry to evaluate the integral, then check your answer with your calculator. Note: Assume $k > 0$ and $r > 0$.

Examples:

1. $\int_{-1}^3 7 \, dx$

4. $\int_{-3}^3 (4 - |x|) \, dx$

2. $\int_1^4 2x \, dx$

5. $\int_0^3 \sqrt{9 - x^2} \, dx$

3. $\int_0^3 (-2x + 6) \, dx$

A new geometric formula!

Archimedes discovered that the area of a parabolic arch can be found using the formula

$$\text{Area} = \frac{2}{3} \times \text{base} \times \text{height}$$

Find the area of the region bounded by $f(x) = 9 - x^2$ and the x -axis.

Problems:

6. $\int_{-5}^5 4 \, dx$

12. $\int_{-2}^2 \sqrt{4 - x^2} \, dx$

7. $\int_a^b k \, dx$

13. $\int_{-k}^k (k - |x|) \, dx$

8. $\int_2^{10} \frac{1}{2}x \, dx$

14. $\int_0^k kx \, dx$

9. $\int_2^{12} (8 - \frac{1}{2}x) \, dx$

15. $\int_0^r \sqrt{r^2 - x^2} \, dx$

10. $\int_{-3}^3 (|x| + 2) \, dx$

16. $\int_0^6 (6x - x^2) \, dx$

11. $\int_{-3}^3 |x + 2| \, dx$

17. $\int_{-\sqrt{k}}^{\sqrt{k}} (k - x^2) \, dx$